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STRUCTURE OF THE CONSTITUTIVE RELATIONS FOR HEREDITARILY ELASTIC MATERIALS REINFORCED BY HARD FIBERS

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The problem of simplifying the nonlinear hereditary elasticity relations is considered for strongly anisotropic materials such as fiber-reinforced composites. This is done using their property that the material stiffness is high along the reinforcement and is low in the cross-sectional direction. The material is considered transversally isotropic. The simplification is performed by analyzing asymptotic representations of creep relations. Relations of various degrees of accuracy for various types of composites and stress states are obtained.

Key words: *hereditary elasticity, nonlinearity, fibrous composite, transversal isotropy, asymptotic analysis.*

There are two basic methods for determining the constitutive relations for composite materials (CMs). In the first method, the mechanical properties of a CM can be determined experimentally on samples whose material is considered homogeneous and anisotropic (the phenomenological approach). This method, however, requires a large number of experiments since the material properties depend not only on the physicomechanical characteristics of the composite phases but also on its structure and manufacturing techniques. In the structural approach, the mechanical characteristics of a CM are determined from the well-known properties of the starting components. A disadvantage of this method is that in compositions, materials often behave absolutely differently than in independent tests. However, both the first and second approaches involve the problem of choosing a structure of the constitutive relations that would lead to a reduction in the necessary volume of experimental information.

The nonlinear elasticity relations for CMs have been studied in a number of papers (see, for example, [1–9]). In the present paper, we consider the problem of simplifying the structure of the nonlinear constitutive relations for hereditarily elastic, transversally isotropic materials such as fiberglass, Plexiglas, and carbon fiber-reinforced plastics. The simplification is performed using technique proposed for the case of nonlinear elasticity proposed in [10–13] and for thin shells in the case of nonlinear viscoelasticity in [14]. However, for CMs in shells of medium thickness and thick shells, the method described in [14], does not provide simple constitutive relations. A modification of this approach is described below.

1. Fiber-reinforced materials can be considered orthotropic; therefore, we use a coordinate system attached to the orthotropy axes with the Ox^1 axis directed along the basic reinforcement by hard fibers and the Ox^2 and Ox^3 axes directed across this direction. For convenience in writing and analyzing the constitutive relations, below we use the following notation for the stress-tensor components (σ) and small strains (ε):

$$\tau^1 = \sigma^{11}, \quad \tau^2 = \sigma^{22}, \quad \tau^3 = \sigma^{33}, \quad \tau^4 = \sigma^{23}, \quad \tau^5 = \sigma^{13}, \quad \tau^6 = \sigma^{12},$$
$$e_1 = \varepsilon_{11}, \quad e_2 = \varepsilon_{22}, \quad e_3 = \varepsilon_{33}, \quad e_4 = 2\varepsilon_{23}, \quad e_5 = 2\varepsilon_{13}, \quad e_6 = 2\varepsilon_{12} \qquad (1.1)$$

(τ^i and e_i are the components of the vectors made up of the stress- and strain-tensor components).

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In the theory of hereditary elasticity, the relationship between the static and kinematic characteristics can be written as

$$e_i = A_{ij}(S^1, S^2, \dots)\tau^j + \int_0^t \frac{\partial H}{\partial \tau^i}\, d\theta, \qquad H = H(t-\theta, S^1, \dots). \tag{1.2}$$

Here H is the creep potential and S^i are invariants obtained by convolution of the stress tensor with the tensors describing the mechanical properties of the material.

To make the constitutive relations simple and convenient for use in seeking material functions and constants from experimental data and in solving problems of designing structural members, it is necessary to analyze relations (1.2) with the aim of reducing the dimension of the functions A_{ij} and H. This can be done using the property of strong anisotropy of fiber-reinforced composites.

Let us consider the loading of a sample by time-independent stresses $\tau^i = \tau_0^i = \mathrm{const}^i$. We make the change of variable $\theta_1 = t - \theta$. After that, differentiating relations (1.2) with respect time t, we obtain (the dot above the variable denotes the derivative with respect to t and the subscript c the creep strain):

$$\dot{e}_i = \dot{e}_i^c = \frac{\partial H(t, S^1, S^2, \dots)}{\partial \tau^i}\bigg|_{\tau^i = \tau_0^i}. \tag{1.3}$$

Along with the stress state τ_0^i, we consider the state $\tau_0^i + d\tau^i$. In this case, the rates $\dot{e}_i^c$ change by the quantity

$$d\dot{e}_i^c = B_{ik}\, d\tau^k, \qquad B_{ik} = \frac{\partial^2 H}{\partial \tau^i\, \partial \tau^k}. \tag{1.4}$$

It is known that in a one-dimensional stress state, an increase in the stress leads to an increase in the creep rate. Extending this to the spatial case, we write

$$d\tau^i\, d\dot{e}_i^c > 0. \tag{1.5}$$

Condition (1.5) can be treated as the stability condition for the material.

The diagonal elements of the matrix $\|B\|$ have a physical meaning and characterize the material viscosity for simple strain states: B_{11} for extension or compression along the fibers, B_{22} and B_{33} for extension or compression across the fibers, B_{44} for shear in the plane perpendicular to the reinforcement direction, etc.

To analyze the structure of the potential H, we use the property of fiber-reinforced materials that the creep compliance in the reinforcement direction is lower than the shear compliance and the creep compliance across the fibers. A similar assumption is made for the increments $d\dot{e}_i^c$ and $d\tau^i$. This implies that

$$B_{11} \ll B_{22}, B_{33}, B_{44}, B_{55}, B_{66}. \tag{1.6}$$

However, the use of only relation (1.6) does not allow us to considerably reduce the number of arguments of the function H. Therefore, we narrow the class of the examined CMs and determine the structure of the potential H for reinforced materials in the form of a braid or a strip, which can be considered transversally isotropic in the cross section perpendicular to the reinforcement.

2. For the examined material, the potential H and, hence, the Hesse matrix $\|B\|$ should depend on the invariants independent of the rotation about the axis Ox^1 and mirror mappings in the plane x^2x^3. As such we can choose the following [12, 13, 15, 16]:

$$\begin{gathered} S_1 = \tau^1, \qquad S_2 = \tau^2 + \tau^3, \qquad S_3 = (\tau^5)^2 + (\tau^6)^2, \\ S_4 = (\tau^2)^2 + (\tau^3)^2 + 2(\tau^4)^2, \qquad S_5 = 2\tau^4\tau^5\tau^6 + \tau^2(\tau^6)^2 + \tau^3(\tau^5)^2. \end{gathered} \tag{2.1}$$

Thus, the function H has six arguments:

$$H = H(t, S_1, S_2, \dots, S_5). \tag{2.2}$$

Next, using assumptions (1.6), we introduce small parameters that characterize the ratios of the creep compliances of the material for various simple loads:

$$\eta^2 \approx \frac{B_{11}}{B_{22}} = \frac{B_{11}}{B_{33}} \ll 1, \qquad \xi^2 \approx \frac{B_{22}}{B_{44}} < 1, \qquad \gamma^2 \approx \frac{B_{44}}{B_{55}} = \frac{B_{44}}{B_{66}} < 1. \tag{2.3}$$

The case $B_{66} = B_{55} < B_{44}$ does not differ in principle from that considered here. The calculations are similar to those given below if instead of (1.1) we introduce the notation

$$e_4 = 2e_{13}, \quad e_5 = 2e_{12}, \quad e_6 = 2e_{23}, \quad \tau^4 = \sigma^{13}, \quad \tau^5 = \sigma^{12}, \quad \tau^6 = \sigma^{23}$$

(here and below, the subscript c is omitted).

Along with the notation introduced above, we use matrix and vector symbols, omitting subscripts and denoting by τ and e the vectors with components (1.1) and by B the matrix $\|B\|$.

Relations (1.4) can be written as

$$d\tau = B^{-1}\, d\dot{e} = D\, d\dot{e}. \tag{2.4}$$

An analysis of (2.4) assuming a low creep compliance along the reinforcement suggests that the diagonal elements of the matrix D also have different orders, namely:

$$\frac{D^{22}}{D^{11}} = \frac{D^{33}}{D^{11}} \approx \eta^2 \ll 1, \qquad \frac{D^{44}}{D^{22}} \approx \xi^2 < 1, \qquad \frac{D^{55}}{D^{44}} = \frac{D^{66}}{D^{44}} \approx \gamma^2 < 1. \tag{2.5}$$

From the material stability conditions (1.5) and assumptions (2.5), it follows that the elements of the symmetric matrix D have different orders of smallness with respect to D^{11} and can be estimated by the relation

$$D \approx D^{11} \left| \begin{matrix} 1 & \eta^m & \eta^m & \eta^p\xi^q & \eta^r\xi^s\gamma^b & \eta^z\xi^t\gamma^h \\ \dots & \eta^2 & \eta^2 & \dots & \dots & \dots \\ \dots & \dots & \eta^2 & \dots & \dots & \dots \\ \dots & \dots & \dots & \eta^2\xi^2 & \dots & \dots \\ \dots & \dots & \dots & \dots & \eta^2\xi^2\gamma^2 & \dots \\ \dots & \dots & \dots & \dots & \dots & \eta^2\xi^2\gamma^2 \end{matrix} \right|. \tag{2.6}$$

Here $m,\ p,\ q,\ r,\ s,\ b,\ z,\ t,$ and $h \geqslant 1$.

From (2.6) it follows that dependences (2.4) can be replaced by approximate relations of various orders of accuracy. Next we consider three versions of simplified relations. In the first relation, omitting quantities of order $O(\eta^m)$ compared to unity, we obtain

$$d\tau^1 \approx D^{11}\, d\dot{e}_1 \qquad \text{or} \quad d\dot{e}_1 = (D^{11})^{-1}\, d\tau^1 = C_{11}(t, S_1, S_2, \dots, S_5)\, d\tau^1. \tag{2.7}$$

Since $d\dot{e}_1$ is the total differential, the strain rate $\dot{e}_1$ should depend only on τ^1. Taking into account the expressions for the invariants (2.1), we infer that $\dot{e}_1$ depends only on two arguments:

$$\dot{e}_1 = \varphi_1(t, S_1) = \varphi_1(t, \tau^1). \tag{2.8}$$

As is evident from (2.8), one can easily obtain $\varphi_1(t, \tau^1)$ by analyzing experimental data on the simple extension of CMs for various load levels and approximate it for any system of functions of time and stress τ^1. In determining the approximation coefficients, it is necessary to ensure that the stability conditions for the material (1.5) be satisfied. This can be achieved by two methods: by choosing special basis functions or by using mathematical programming methods with minimization of the residual of calculation and experimental strains subject to constraints (1.5).

The structure of the function $\varphi_1(t, \tau^1)$ can be specified, for example, as a generalization of the relations of the known linear creep kernels. In particular, the creep of many materials, as noted in [17], is adequately described by Abel's kernel. Then, for the linear case of one-dimensional extension–compression problems, relation (2.8) becomes

$$\dot{e}_1 = C(t - \theta)^\beta, \qquad -1 < \beta < 0, \quad C > 0. \tag{2.9}$$

For the nonlinear case, it can be assumed, for example, that $\varphi_1(t, \tau^1)$ has the same structure but the coefficients are related to τ^1, in particular, by the formulas

$$C = (C_0 + C_1\tau^1 + C_2(\tau^1)^2 + \dots)^{2n}; \tag{2.10}$$

$$\beta = -1/(1 + (\beta_0 + \beta_1\tau^1 + \beta_2(\tau^1)^2 + \dots)^{2m})^p. \tag{2.11}$$

This form of representation of C and β ensures that conditions (2.9) are satisfied. In order that conditions (1.5) be satisfied, they need to be written in a certain working range of stresses τ^1 and used as constraints in choosing the sought coefficients by mathematical programming methods. In [18], this approach (with $n = 1$, $m = 1$, and $p = 0.5$) was employed to describe nonlinear creep for multistage loading of Plexiglas cylindrical shells produced by winding of composite plaits. The other linear hereditary relations can be generalized similarly, assuming that the parameters of the employed creep kernels are stress functions.

To determine the structure of the strain rates $\dot{e}_2$ and $\dot{e}_3$, in relations (2.4) for $d\tau^2$ and $d\tau^3$ we retain the first three terms. In view of (2.6), we can write

$$\begin{aligned} d\tau^2 &\approx (\tilde{D}^{21}\eta^m\,d\dot{e}_1 + \tilde{D}^{22}\eta^2\,d\dot{e}_2 + \tilde{D}^{23}\eta^2\,d\dot{e}_3)D^{11}, \\ d\tau^3 &\approx (\tilde{D}^{31}\eta^m\,d\dot{e}_1 + \tilde{D}^{32}\eta^2\,d\dot{e}_2 + \tilde{D}^{33}\eta^2\,d\dot{e}_3)D^{11}. \end{aligned} \tag{2.12}$$

Here $\tilde{D}^{ij} = D^{ij}/D^{11}$, and it can be assumed that $m > 2$. Indeed, from a physical point of view, the change in the stress $d\tau^2$ due to a change in the creep rate $d\dot{e}_2$ across the fibers should be greater than that due to the same change in $d\dot{e}_1$ in the longitudinal direction. This is supported by the following calculations.

Let us solve Eqs. (2.7) and (2.12) for $d\dot{e}_2$ and $d\dot{e}_3$:

$$\begin{aligned} d\dot{e}_2 &= d\tau^1 B_{21}(t, S_1, S_2, \ldots, S_5) + d\tau^2 B_{22}(t, S_1, S_2, \ldots, S_5) + d\tau^3 B_{23}(t, S_1, S_2, \ldots, S_5), \\ d\dot{e}_3 &= d\tau^1 B_{31}(t, S_1, S_2, \ldots, S_5) + d\tau^2 B_{32}(t, S_1, S_2, \ldots, S_5) + d\tau^3 B_{33}(t, S_1, S_2, \ldots, S_5). \end{aligned} \tag{2.13}$$

According to relations (1.3), the following condition should be satisfied:

$$\frac{\partial \dot{e}_i}{\partial \tau_k} = \frac{\partial \dot{e}_k}{\partial \tau_i}. \tag{2.14}$$

Because $\dot{e}_1$ does not depend on τ^2 and τ^3, from (2.14) it follows that

$$B_{21} = 0, \qquad B_{31} = 0. \tag{2.15}$$

Solving now system (2.13) for the stress increments, we find that they depend only on de^2 and de^3. Hence, in expressions (2.12), the first terms should not be taken into account by virtue of the adopted assumptions. This implies the smallness of the functions $\tilde{D}^{21}$ and $\tilde{D}^{31}$ or the smallness of the factors η^m. Since they appear as cofactors, we assume that $\eta^m \ll \eta^2$, i.e., $m > 2$.

An analysis of relations (2.13) also shows that according to (2.14) and taking into account the adopted assumptions, the parameters $\dot{e}_2$ and $\dot{e}_3$ should not depend on τ^4, τ^5, and τ^6. In turn, parameters $\dot{e}_4$, $\dot{e}_5$, and $\dot{e}_6$ should depend on τ^1, τ^2, and τ^3.

For the further concrete definition of the constitutive relations, we take into account that $\dot{e}_i$ are expressed in terms of the potential H, which is a function of the invariants (2.1). Then, the expressions for $\dot{e}_i$ are written as

$$\dot{e}_1 = \frac{\partial H}{\partial \tau^1} = \frac{\partial H}{\partial S_1}; \tag{2.16}$$

$$\dot{e}_2 = \frac{\partial H}{\partial \tau^2} = \frac{\partial H}{\partial S_2} + \frac{\partial H}{\partial S_4}\,2\tau^2 + \frac{\partial H}{\partial S_5}\,(\tau^6)^2; \tag{2.17}$$

$$\dot{e}_3 = \frac{\partial H}{\partial \tau^3} = \frac{\partial H}{\partial S_2} + \frac{\partial H}{\partial S_4}\,2\tau^3 + \frac{\partial H}{\partial S_5}\,(\tau^5)^2; \tag{2.18}$$

$$\dot{e}_4 = \frac{\partial H}{\partial S_4}\,4\tau^4 + \frac{\partial H}{\partial S_5}\,2\tau^5\tau^6; \tag{2.19}$$

$$\dot{e}_5 = \frac{\partial H}{\partial S_3}\,2\tau^5 + \frac{\partial H}{\partial S_5}\,2(\tau^3\tau^5 + \tau^4\tau^6); \tag{2.20}$$

$$\dot{e}_6 = \frac{\partial H}{\partial S_3}\,2\tau^6 + \frac{\partial H}{\partial S_5}\,2(\tau^2\tau^6 + \tau^4\tau^5). \tag{2.21}$$

Because parameters $\dot{e}_2$ and $\dot{e}_3$ do not depend on τ^5 and τ^6 and parameters $\dot{e}_5$ and $\dot{e}_6$ do not depend on τ^2 and τ^3, from (2.16)–(2.21) it follows that $\partial H/\partial S_5 = 0$.

Next, we subtract (2.18) from (2.17):

$$\dot{e}_2 - \dot{e}_3 = \frac{\partial H}{\partial S_4}\,2(\tau^2 - \tau^3). \tag{2.22}$$

Because $\dot{e}_2$ and $\dot{e}_3$ do not depend on τ^4 and $\dot{e}_4$ does not depend on τ^2 and τ^3, a comparison of (2.22) and (2.19) shows that $\partial H/\partial S_4$ does not depend on τ^i. We denote

$$\frac{\partial H}{\partial S_4} = H_{44}(t). \tag{2.23}$$

Then, from relations (2.14) it follows that $\dot{e}_5$ and $\dot{e}_6$ should depend not only on τ_1, τ_2, and τ_3 but also on τ_4. This implies that $\partial H/\partial S_3$ does not depend on S_1, S_2, and S_4. In view of the aforesaid, H can be written as

$$H = H_1(t,\tau^1) + H_2(t,S_2) + H_{44}(t)(\tau^4)^2 + H_3(t,S_3). \tag{2.24}$$

Thus, in the simplest case, the potential H for a transversally isotropic material contains three functions of two arguments and one function of one argument instead of a function of six arguments. The functions H_2, H_{44}, and H_3 can be determined from simple experiments with extension–compression across the reinforcement and shears in the planes (x^1, x^2) and (x^2, x^3). If such tests are technically difficult, these functions are determined by solving some inverse problems of the mechanics of structures (using so-called identification methods).

The particular form of the functions H_1, H_2, and H_3 can be chosen in a similar manner as was done for the functions φ_1 in formula (2.8), for example, by generalizing the linear Hereditary relations. In particular, if the initial creep kernel is Abel's kernel, they can be written as

$$H_1 = \int (C_{10} + C_{11}\tau^1 + C_{12}(\tau^1)^2 + \ldots)^{2n_1} t^{-1/(1+(\beta_{10}+\beta_{11}\tau^1+\ldots)^{2m_1})^{p_1}}\, d\tau^1; \tag{2.25}$$

$$H_2 = \int (C_{20} + C_{21}S_2 + C_{22}(S_2)^2 + \ldots)^{2n_2} t^{-1/(1+(\beta_{20}+\beta_{21}S_2+\ldots)^{2m_2})^{p_2}}\, dS_2; \tag{2.26}$$

$$H_3 = \int (C_{30} + C_{31}S_3 + C_{32}(S_3)^2 + \ldots)^{2n_3} t^{-1/(1+(\beta_{30}+\beta_{31}S_3+\ldots)^{2m_3})^{p_3}}\, dS_3.$$

We recall that in determining C_{ij} and β_{ij}, it is necessary to ensure that conditions (1.5) are satisfied in the working range of stresses.

Next, we consider the second version of the nonlinear hereditary relations, which can be called refined relations. Retaining the first three terms in relations (2.4) for $d\tau^1$, $d\tau^2$, and $d\tau^3$ and performing an analysis as was done above, we write the potential H as

$$H = H_1(t,\tau^1,S_2) + H_{44}(t)(\tau^4)^2 + H_3(t,S_3). \tag{2.27}$$

In this case, to determine H_1, it is necessary to perform an experiment with a biaxial stress state for various load levels.

Reducing the accuracy of the representation of the increments in the linear strain rate $d\dot{e}_i$ in (2.4), we can write H_1 as the sum of functions of two rather than three arguments and obtain something intermediate between the elementary representation of H in the form of (2.24) and the refined representation in the form of (2.27). For this, we assume that in relations (2.4), the functions D^{12} and D^{13} are equal to certain averaged values, i.e., as in linear heredity theory, they do not depend on S_1 and S_2. The reason for such simplification can be the fact that D_{12}, D_{13}, and D_{23} are smaller than D_{11}. Then, in view of the transversal isotropy properties we obtain

$$H_1 = H_{11}(t,\tau^1) + H_{12}(t)\tau^1 S_2 + H_{22}(t,S_2). \tag{2.28}$$

For identification from experiments, the functions H_{11} and H_{22} can be specified, for example, in the form of (2.25) and (2.26), respectively.

Relations (2.24) and (2.27) have one disadvantage: an analysis of the constitutive relations shows that in the plane of isotropy, the shear strain rate depends linearly on the shear stress with a certain degree of accuracy. However, if the fibers are thin and hard and their volume fraction is not too large, the strain of the composite is determined primarily by the properties of the matrix. Since, in most cases, the matrix can be considered isotropic, in the case of extension–compression and shear in the plane of isotropy, as in the case of nonlinear elasticity, the nonlinearity level will most likely be identical. Thus, we obtain constitutive relations that take into account this fact, which will be called the third version of the simplified creep relations. For this, in expressions (2.4) we retain only the first term for $d\tau^1$ and four rather than three terms for $d\tau^2$ and $d\tau^3$. Then, just as was done above, we arrive at the expression

$$H = H_1(t,\tau^1) + H_{24}(t,S_2,S_4) + H_3(t,S_3). \tag{2.29}$$

Experimental determination of the function $H_{24}(t, S_2, S_4)$ is a complex problem. Its successful solution, on the one hand, depends on the choice of the form of H_{24}. On the other hand, the problem is complicated by the fact that conditions (1.5) should be satisfied in the working range of the parameters S_2, S_3, and S_4. Since it is now necessary to write these conditions for various combinations of three variables, the number of restrictions in the mathematical programming problem increases considerably. The solution of this problem can be facilitated as was done in [19]; i.e., in determining the structure of the stiffness characteristics of a nonlinearly elastic, transversally isotropic material in the plane of isotropy, it is possible to use the constitutive relations for an isotropic material taking into account the smallness of the longitudinal (along the reinforcement) strains. Here we assume that in the absence of stresses τ^1, the hereditary relations in the plane of isotropy should be similar to the relations for an isotropic material. Then, relations (2.29) can be written in simplified form

$$H = H_1(t, \tau^1) + H_0(t, \sigma_i),$$

where σ_i is the stress intensity for $\tau^1 = 0$. For our case, it has the form

$$\sigma_i^2 = (\tau^2)^2 + (\tau^3)^2 - (\tau^2\tau^3) + 3(\tau^4)^2 + 3(\tau^5)^2 + 3(\tau^6)^2 = 0.5(3S_4 - S_2^2 + 6S_3).$$

We can now write the function $H_0(t, \sigma_i)$ in a form similar to (2.25), i.e., set σ_i instead of the argument τ^1. The satisfaction of condition (1.5) is also strongly simplified since it is reduced to the condition $\partial^2 H_0/\partial\sigma_i\,\partial\sigma_i > 0$.

As in the previous case, relations (2.29) can be supplemented by one more function of a single variable that corresponds to linear heredity and takes into account the effect of τ^2 and τ^3 on $\dot{e}_1$. In this case, H becomes

$$H = H_1(t, \tau^1) + H_{12}(t)\tau^1 S_2 + H_{24}(t, S_2, S_4) + H_3(t, S_3). \quad (2.30)$$

As can be seen from (2.29) and (2.30), the effect of shear stresses on the linear strain rate in the plane of isotropy is taken into account in this case.

The above relations can also be used to analyze shells of medium thickness using the hypothesis $\sigma^{33} = \tau^3 = 0$. In this case, $\dot{e}_3 \neq 0$ but the contribution of this strain rate to the energy is equal to zero and there is no need to know its dependence on stress.

Thus, accounting for the behavior of fibrous composites makes it possible to simplify and obtain consistent forms of the nonlinear hereditary elasticity relations for fiber-reinforced materials. The errors of the proposed simplified constitutive relations, for example, the assumptions of the nondependence of the linear strain rate on the shear stress in the form of (2.8) and (2.24), can be estimated for particular materials only from experimental studies of composites.

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